Random Variable and Probability Distributions

Random Variable

A random variable (r.v) is a real valued function defined over the Sample Space. So its domain of definition is the sample space S and range is the real line extending from $-\infty$ to $+\infty$. In other words, a r.v is a mapping from sample space to real numbers. Random variables are also called chance variables or Stochastic variables. It is denoted by X or $X(\omega)$.

In symbols, $X : S \to R(-\infty, +\infty)$

For example, consider a random experiment consisting of two tosses of a coin. Then the sample space is given by $S = \{HH, HT, TH, TT\}$

Then to each outcome ω in the sample space there corresponds a real number $X(\omega)$ - here it represents the number of heads obtained. It can be presented in the tabular form as,

Outcome (ω):	HH	HT	TH	TT
Values of $X(\omega)$:	2	1	1	0

Example:

In coin tossing experiment, we note that

$S = \{\omega_1, \omega_2\}$ where ω_1 = Head, ω_2 = Tail

Now define $X(\omega) = \begin{cases} 0, & \text{if } \omega = \text{Tail (T)} \\ 1, & \text{if } \omega = \text{Head (H)} \end{cases}$

Here the random variable $X(\omega)$ takes only two values as ω can be either head or tail.

Note: If X_1 and X_2 are r.v.s and C is a Constant then,

i) CX_1 is a r.v

ii) $X_1 + X_2$ is a r.v

iii) $X_1 - X_2$ is a r.v

iv) $\max[X_1, X_2]$ is a r.v

v) $\min[X_1, X_2]$ is a r.v

Random Variables are of two types:

i) Discrete ii) Continuous

A random variable X is said to be discrete if its range includes finite number of values or countably infinite number of values.

The possible values of a discrete r.v can be labelled as $x_1, x_2, x_3 \ldots$

eg: No. of defective articles produced in a factory in a day in a city.

A random variable which is not discrete is said to be Continuos. That means it can assume infinite number of values from a specified interval of the form $[a, b]$

eg. X represents the service time of a doctor on his next patient.

Note that Random Variables are denoted by Capital letters X, Y, Z etc. and the corresponding small letters are used to denote the value of a random variable.

Probability Distributions

By probability Distribution of a random variable X we mean the assignment of probabilities to all events defined in terms of this random variable.

i) Discrete :

The probability distribution or simply distribution of a discrete r.v is a list of the distinct values of x_i of X with their associated probabilities $f(x_i) = P(X = x_i)$

Thus let X be a discrete r.v assuming the values $x_1, x_2 \cdots x_n$ from the real line. Let the corresponding probabilities be $f(x_1)$ $f(x_2) \ldots f(x_n)$. Then $P(X = x_i) = f(x_i)$ is called probability mass function or probability function of X, provided it satisfy the conditions

i) $f(x_i) \geq 0$ for all i

ii) $\Sigma f(x_i) = 1$

The probability distribution of x may be either in the form of a table or in the form of a formula.

ii) Continuos:

If a random variable is a Continuos Variable, its probability distribution is called a Continuos probability distribution.

A Continuos probability distribution cannot be expressed in tabular form. Instead, an equation or formula is used to describe a continuos probability distribution.

If x is a Continuos r.v and if $P(x \le X \le x+dx) = f(x)dx$, then $f(x)$ is called probability density function (P.d.f) of a Continuos r.v provided it satisfy the conditions

i) $f(x) \ge 0 \;\forall x$ ii) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

Result 1: $P(a<x<b) = P(a<x \le b) = P(a \le x < b)$

$= P(a \le x \le b) = \int_a^b f(x)\,dx$

= area under the curve $y = f(x)$ enclosed b/w the co-ordinates drawn at $x = a$ and $x = b$.

Result 2: Probability of a Continuos r.v X will assume a particular value is zero i.e, $P(X=a) = 0$